

Anavrinisis EE α'δροισγα - γινόγενο πιστών.

1. a) $x_1 + x_2 = -3$, $x_1 x_2 = -5$ b) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 9 + 10 = 19$

c) $x_1^3 + x_2^3 = (x_1 + x_2)(x_1^2 - x_1 x_2 + x_2^2) = -3 \cdot (19 + 5) = -72$

d) $\Delta v \quad \alpha = |x_1 - x_2| \Rightarrow \alpha^2 = x_1^2 + x_2^2 - 2x_1 x_2 = 19 + 10 = 29 \Rightarrow \alpha = \sqrt{29}$

e) $x_1^4 + x_2^4 = (x_1^2 + x_2^2)^2 - 2(x_1 x_2)^2 = 19^2 - 2 \cdot 25 = 361 - 50 = 311$

f) $|x_1^4 - x_2^4| = |x_1^2 - x_2^2| \cdot (x_1^2 + x_2^2) = |x_1 - x_2| \cdot |x_1 + x_2| \cdot 19 = 57 \cdot \sqrt{29}$

2. a) $x_1 + x_2 = 1$, $x_1 x_2 = -3$ b) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 1 + 6 = 7$

c) $x_1^3 x_2 + x_1 x_2^3 = x_1 x_2 (x_1 + x_2) = -3$

d) $\Delta v \quad \alpha = |x_1 - x_2| \Rightarrow \alpha^2 = x_1^2 + x_2^2 - 2x_1 x_2 = 7 + 6 = 13 \Rightarrow \alpha = \sqrt{13}$

e) $x_1^4 x_2^2 + x_2^4 \cdot x_1^4 = x_1^2 x_2^2 \cdot (x_1^2 + x_2^2) = 9 \cdot 7 = 63$

3. a) $x_1 + x_2 = 1$, $x_1 x_2 = -12 \rightarrow x^2 - x - 12 = 0$

b) $x_1 + x_2 = -3$, $x_1 x_2 = -10 \rightarrow x^2 + 3x - 10 = 0$

c) $x_1 + x_2 = -\frac{2}{3} + 2 = \frac{4}{3}$, $x_1 x_2 = -\frac{4}{3} \rightarrow x^2 - \frac{4}{3}x - \frac{4}{3} = 0 \Rightarrow 3x^2 - 4x - 4 = 0$

d) $x_1 + x_2 = \frac{\alpha^2 + 6}{2\alpha}$, $x_1 x_2 = \frac{3}{2} \rightarrow x^2 - \frac{\alpha^2 + 6}{2\alpha}x + \frac{3}{2} = 0 \Rightarrow 2\alpha x^2 - (\alpha^2 + 6)x + 3\alpha = 0$

e) $x_1 + x_2 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, $x_1 x_2 = -\frac{1}{6} \rightarrow x^2 - \frac{1}{6}x - \frac{1}{6} = 0 \Rightarrow 6x^2 - x - 1 = 0$

f) $x_1 + x_2 = 4$, $x_1 x_2 = 1 \rightarrow x^2 - 4x + 1 = 0$

4. Είρων $x_1 + x_2 = -5$, $x_1 x_2 = 3$. a) $2x_1 - 3 + 2x_2 - 3 = 2(x_1 + x_2) - 6 = -16$

$(2x_1 - 3)(2x_2 - 3) = 4x_1 x_2 - 6(x_1 + x_2) + 9 = 12 + 30 + 9 = 51 \rightarrow x^2 + 16x + 51 = 0$

b. $S = x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 25 - 6 = 19$ $\rho = (x_1 x_2)^2 = 9 \rightarrow x^2 - 19x + 9 = 0$

c. $S = \frac{x_1}{x_2^2} + \frac{x_2}{x_1^2} = \frac{x_1^3 + x_2^3}{(x_1 x_2)^2} = \frac{(x_1 + x_2)(x_1^2 - x_1 x_2 + x_2^2)}{(x_1 x_2)^2} = \frac{-5 \cdot (19 - 3)}{9} = -\frac{80}{9}$

$\rho = \frac{x_1}{x_2^2} \cdot \frac{x_2}{x_1^2} = \frac{1}{x_1 x_2} = \frac{1}{3} \rightarrow x^2 + \frac{80}{9}x + \frac{1}{3} = 0 \Rightarrow 9x^2 + 80x + 3 = 0$

5. a) $x^2 - 10x + 16 = 0 \Rightarrow x_1 = 2$, $x_2 = 8$

b) $x^2 - 3x + 2 = 0 \Rightarrow x_1 = 1$, $x_2 = 2$

c) $x^2 - 3x + 1 = 0 \Rightarrow x_1 = \frac{3 - \sqrt{5}}{2}$, $x_2 = \frac{3 + \sqrt{5}}{2}$

6. Einde $x_1 = 2x_2$, $x_1 + x_2 = 12$, $x_1 x_2 = k$ dpa $3x_2 = 12 \Rightarrow x_2 = 4$
dpa $x_1 = 8$ uan $x_1 x_2 = k = 32$, uan unäpravur yiqzi $\Delta = 144 - 128 = 16$

7. $x_1 = 2x_2$ }
 $x_1 + x_2 = k^2 - 1$ }
 $x_1 x_2 = 8$ } $2x_2^2 = 8 \Rightarrow x_2 = \pm 2$ dpa $x_1 = \pm 4$ suvgnis:

$$x_1 + x_2 = 6 = k^2 - 1 \Rightarrow k^2 = 7 \Rightarrow k = \pm \sqrt{7}$$

f $x_1 + x_2 = -6 = k^2 - 1$, adurazn. Teda, $x_1 = 4$, $x_2 = 2$, $k = \pm \sqrt{7}$

8. $x_1 = x_2^2$, $x_1 + x_2 = k^2 + 2$, $x_1 x_2 = -8 \Rightarrow x_2^3 = -8 \Rightarrow x_2 = -2$ dpa $x_1 = 4$
suvgnis npene $k^2 + 2 = -8 \Rightarrow k^2 = -10$, adurazn.

9. a) Préle $s=0 \Rightarrow k=3$ suvgnis: $x^2 - 4 = 0 \Rightarrow x = 2$ n $x = -2$

b) Préle $k \neq 0$ uan $k^2 - 3k = 0 \Rightarrow k=3$, suvgnis: $3x^2 - 12 = 0 \Rightarrow x = \pm 2$.

10. a) Préle $k \neq 0$ uan $\frac{k^2 + k - 4}{k} = 1 \Rightarrow k^2 = 4 \Rightarrow k=2$ n $k=-2$

da $k=2$: $2x^2 - 5x + 2 = 0$ dpa $x_1 = 2$, $x_2 = \frac{1}{2}$

da $k=-2$: $-2x^2 - 5x - 2 = 0 \Rightarrow 2x^2 + 5x + 2 = 0$ dpa $x_1 = -2$, $x_2 = -\frac{1}{2}$

b) Préle $k^2 - 3k + 1 = 1 \Rightarrow k=0$ n $k=3$

da $k=0$: $x^2 - x + 1 = 0$, adurazn.

da $k=3$: $x^2 - 4x + 1 = 0$ dpa $x_1 = 2 - \sqrt{3}$, $x_2 = 2 + \sqrt{3}$

11. a) $\Delta = k^2 - 4(k-1) = k^2 - 4k + 4 = (k-2)^2 \geq 0$

b) Da spene $s < 0$, $p > 0 \Rightarrow k < 0$ uan $k-1 > 0 \Rightarrow k < 0$ uan $k > 1$, adurazn.

c) Préle $k-1 < 0 \Rightarrow k < 1$.

12. a) $\Delta = (2k+1)^2 - 8k = 4k^2 + 4k + 1 - 8k = 4k^2 - 4k + 1 = (2k-1)^2 \geq 0$

b) $A = x_1 x_2 (x_1 + x_2) - 3x_1 x_2 - \Delta = (2k+1) \cdot 2k - 6k - (4k^2 - 4k + 1) = -1$

c) Préle $2k-1 = 1 \Rightarrow k = \frac{1}{2}$ dpa n sigras: $x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$ dpa $x_1 = x_2 = 1$.