

Αναγνώριση ΕΠ3-2223

ΘΓ. Γ1. Έστω $x_1 < x_2 \Rightarrow -x_1 > -x_2 \Rightarrow e^{-x_1} > e^{-x_2} \Rightarrow$
 $\Rightarrow \frac{1}{x_1} > \frac{1}{x_2}$

$f(x_1) > f(x_2)$ άρα $f \downarrow$.

Ένε.δ. $\lim_{x \rightarrow 0^+} f(x) = +\infty$ και $\lim_{x \rightarrow +\infty} (e^{-x} + \frac{1}{x} - 2) = -2$

δ.ν. σύνολο τιμών = $(-2, +\infty)$.

Γ2. Από το $0 \in (-2, +\infty)$, (Θετ) $\exists x_0 \in (0, +\infty) : f(x_0) = 0$

Γ3. Η επίσημη είναι αδύνατη αν $|nk| \leq -2 \Rightarrow k \leq \frac{1}{e^2} \Rightarrow$
 $ke^2 - 1 \leq 0$, οπότε $\lim_{x \rightarrow +\infty} e^{(ke^2 - 1)x} = \lim_{y \rightarrow -\infty} e^y = 0$, για $k < \frac{1}{e^2}$
και 1 για $k = \frac{1}{e^2}$

ΘΔ. $f^2(x) - 4f(x)\cos x + 2 = x^2 + 4\sin^2 x$, $f(0) = 2 - \sqrt{2}$.

$f^2(x) - 4\cos x f(x) + 4\cos^2 x + 2 = x^2 + 4\sin^2 x + 4\cos^2 x \Rightarrow$

$(f(x) - 2\cos x)^2 = x^2 + 2 \Rightarrow |f(x) - 2\cos x| = \sqrt{x^2 + 2}$

Αν $g(x) = f(x) - 2\cos x$, $g(x) \neq 0$, $g(x)$ συνεχής και

$g(0) = 2 - \sqrt{2} - 2 = -\sqrt{2} < 0$ άρα $g(x) < 0$ άρα

$-f(x) + 2\cos x = \sqrt{x^2 + 2} \Rightarrow f(x) = 2\cos x - \sqrt{x^2 + 2}$

Δ2. $2\cos x - \sqrt{x^2 + 2} \leq 0 \Rightarrow 2\cos x \leq \sqrt{x^2 + 2} \Rightarrow$

$4\cos^2 x \leq x^2 + 2$. $x^2 + 2 \geq 2$, $2\cos x \leq 2$
 $-\sqrt{x^2 + 2} \leq -\sqrt{2}$

$f(x) \leq 2 - \sqrt{2} \quad \forall x \in \mathbb{R}$

Δ3. $\lim_{x \rightarrow +\infty} (2\cos x - \sqrt{x^2 + 2}) = \lim_{x \rightarrow +\infty} x \left(\frac{2\cos x}{x} - \sqrt{1 + \frac{2}{x^2}} \right) = -\infty$

$\lim_{x \rightarrow +\infty} \frac{2\cos x}{x} - \frac{x \sqrt{1 + \frac{2}{x^2}}}{x} = 0 - 1 = -1$.

14. $f(x+lnx) \leq f(x) \iff x+lnx \geq 1$

$h(x) = x + \ln x - 1$, $h'(x) = 1 + \frac{1}{x}$

$h(1) = 0$, $h(x) \geq 0 \iff h'(x) = 1 + \frac{1}{x} > 0$

$x \geq 1$

[Faint handwritten notes and calculations, including a boxed 'x >= 1' and some derivative work.]

[Faint handwritten notes, possibly related to limits or function analysis.]

[Faint handwritten notes, possibly related to function analysis.]

[Faint handwritten notes, possibly related to function analysis.]

[Faint handwritten notes, possibly related to function analysis.]