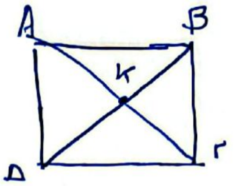
Test 1



A1 : Σ - Σ - Σ - Σ - Σ

A2 : Η διαγώνιος έχει μήκος $4\sqrt{2}$.

Συνεπώς $\vec{AB}\cdot\vec{AK} = 4\cdot 2\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 8$ $\quad$ $\vec{A\Delta}\cdot\vec{A\Gamma} = 4\cdot 4\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 16$

$\vec{AB}\cdot\vec{\Delta\Gamma} = 4\cdot 4\cdot 1 = 16$ $\quad$ $\vec{B\Gamma}\cdot\vec{A\Gamma} = 4\cdot 4\sqrt{2}\cdot \text{συν}45 = 16$

ΘΒ : $A(4,-3)$ $\ B(6,-2)$ $\ \Gamma(7,-4)$. $\ \vec{\alpha} = \vec{AB} = (2,1)$ $\ \vec{\beta} = \vec{A\Gamma} = (3,-1)$ $\ \vec{B\Gamma} = (1,-2)$

B1. Επειδή $\det(\vec{AB},\vec{A\Gamma}) = -5 \neq 0$, τα A, B, Γ όχι συνευθειακά.

Επίσης, $\vec{AB}\cdot\vec{B\Gamma} = (2,1)\cdot(1,-2) = 0$ άρα $\vec{AB}\perp\vec{B\Gamma} \Rightarrow \hat{B} = 90^\circ$

B2. Έστω $\varphi = \hat{A} = (\vec{AB},\vec{A\Gamma})$. Είναι $\text{συν}\varphi = \dfrac{\vec{AB}\cdot\vec{A\Gamma}}{|\vec{AB}|\cdot|\vec{A\Gamma}|} = \dfrac{6-1}{\sqrt{5}\cdot\sqrt{10}} = \dfrac{5}{5\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

συνεπώς $\varphi = 45^\circ$.

B3. Έστω $\vec{v} = (x,y)$. Τότε $\vec{v}+\vec{\beta} = (x+3, y-1) \parallel \vec{\alpha} \Rightarrow \begin{vmatrix} x+3 & y-1 \\ 2 & 1 \end{vmatrix} = 0 \Rightarrow$

$x+3-2y+2 = 0 \Rightarrow x = 2y-5$.

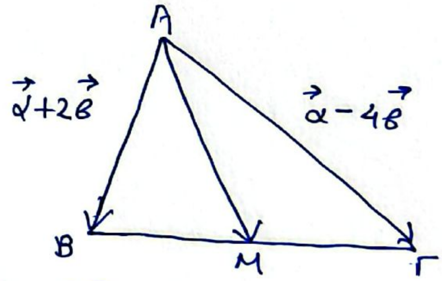
Επίσης, $3\vec{\alpha}-\vec{v} = (6,3)-(x,y) = (6-x, 3-y) \perp \vec{\alpha}+\vec{\beta} = (5,0)$, συνεπώς

$(3\vec{\alpha}-\vec{v})\cdot(\vec{\alpha}+\vec{\beta}) = 0 \Rightarrow 5(6-x) = 0 \Rightarrow x = 6$ άρα $y = 11/2$ δηλ. $\vec{v} = (6, 11/2)$

ΘΓ : Γ1: $\vec{\alpha}\vec{\beta} = |\vec{\alpha}|\cdot|\vec{\beta}|\cdot\text{συν}60^\circ = 2\cdot 1\cdot\frac{1}{2} = 1$

$\vec{AM} = \dfrac{\vec{AB}+\vec{A\Gamma}}{2} = \dfrac{2\vec{\alpha}-2\vec{\beta}}{2} = \vec{\alpha}-\vec{\beta}$

και $\vec{B\Gamma} = \vec{A\Gamma} - \vec{AB} = -6\vec{\beta}$



Γ2. $AM = |\vec{AM}| = |\vec{\alpha}-\vec{\beta}|$, συνεπώς $|\vec{AM}|^2 = \vec{\alpha}^2+\vec{\beta}^2-2\vec{\alpha}\vec{\beta} = 4+1-2 = 3 \Rightarrow |\vec{AM}| = \sqrt{3}$

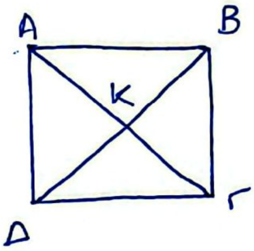
$AB = |\vec{AB}| = |\vec{\alpha}+2\vec{\beta}|$ άρα $|\vec{AB}|^2 = \vec{\alpha}^2+4\vec{\beta}^2+4\vec{\alpha}\vec{\beta} = 12 \Rightarrow |\vec{AB}| = 2\sqrt{3}$.

Γ3. Έστω $\varphi = (\widehat{\vec{AB},\vec{AM}})$. Είναι $\vec{AB}\cdot\vec{AM} = \vec{\alpha}^2 - 2\vec{\beta}^2 + \vec{\alpha}\vec{\beta} = 4-2+1 = 3$

συνεπώς $\text{συν}\varphi = \dfrac{\vec{AB}\cdot\vec{AM}}{|\vec{AB}|\cdot|\vec{AM}|} = \dfrac{3}{2\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{2}$ δηλ. $\varphi = 60^\circ$.

Test 2

A1: Λ - Σ - Σ - Σ - Σ



A2: Η διαγώνιος έχει μήκος ίσο με $2\sqrt{2}$.

$\vec{A\Delta}\cdot\vec{AK} = 2\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 2$

$\vec{AB}\cdot\vec{\Gamma\Delta} = -4$ $\quad \vec{AB}\cdot\vec{A\Gamma} = 2\cdot 2\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 4$

$\vec{B\Gamma}\cdot\vec{A\Gamma} = \vec{A\Delta}\cdot\vec{A\Gamma} = 2\cdot 2\sqrt{2}\cdot\frac{\sqrt{2}}{2} = 4.$

ΘΒ. $A(7,-4)$, $B(4,-3)$, $\Gamma(6,-2)$. Άρα $\vec{\alpha} = \vec{AB} = (-3,1)$

$\vec{\beta} = \vec{B\Gamma} = (2,1)$, $\vec{A\Gamma} = (-1,2)$.

B1. Είναι $\det(\vec{AB},\vec{B\Gamma}) = -5 \neq 0$ δηλ. τα A, B, Γ όχι συνευθειακά.
Επίσης $\vec{B\Gamma}\cdot\vec{A\Gamma} = 0$ δηλ. $\vec{B\Gamma}\perp\vec{A\Gamma}$ άρα $\hat{\Gamma} = 90^\circ$

B2. $\sigma\upsilon\nu B = \sigma\upsilon\nu(\vec{BA},\vec{B\Gamma}) = \frac{\vec{BA}\cdot\vec{B\Gamma}}{|\vec{BA}|\cdot|\vec{B\Gamma}|} = \frac{(3,-1)\cdot(2,1)}{\sqrt{10}\cdot\sqrt{5}} = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}$, άρα
$\hat{B} = 45^\circ$

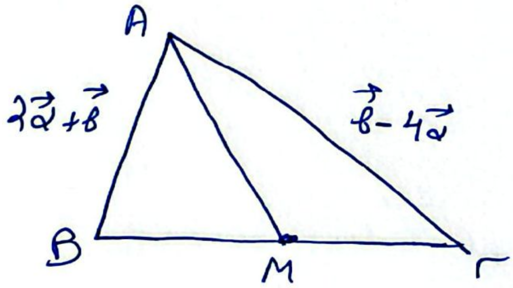
B3. Έστω $\vec{v} = (x,y)$. Είναι $\vec{v}-\vec{\beta} = (x-2,y-1)$, $\vec{\alpha}-\vec{\beta} = (-5,0)$
και $2\vec{\alpha}-\vec{v} = (-6,2)-(x,y) = (-6-x,2-y)$.

$(\vec{v}-\vec{\beta})\parallel\vec{\alpha} \iff \begin{vmatrix} x-2 & y-1 \\ -3 & 1 \end{vmatrix} = 0 \Rightarrow x-2+3y-3 = 0 \Rightarrow x = 5-3y$

και $(2\vec{\alpha}-\vec{v})\perp(\vec{\alpha}-\vec{\beta}) \iff (-6-x,2-y)\cdot(-5,0) = 0 \Rightarrow x = -6$

άρα $y = 11/3$

δηλ. $\vec{v} = (-6, 11/3)$.



ΘΓ. Είναι $\vec{\alpha}\vec{\beta} = 1\cdot 2\cdot\frac{1}{2} = 1$

$\vec{AM} = \frac{\vec{AB}+\vec{A\Gamma}}{2} = \frac{2\vec{\beta}-2\vec{\alpha}}{2} = \vec{\beta}-\vec{\alpha}$

$\vec{B\Gamma} = \vec{A\Gamma}-\vec{AB} = -6\vec{\alpha}$.

Γ2. $|\vec{AM}|^2 = \vec{\beta}^2+\vec{\alpha}^2-2\vec{\alpha}\vec{\beta} = 4+1-2 = 3 \Rightarrow |\vec{AM}| = \sqrt{3}$

$|\vec{AB}|^2 = 4\vec{\alpha}^2+\vec{\beta}^2+4\vec{\alpha}\vec{\beta} = 4+4+4 = 12 \Rightarrow |\vec{AB}| = 2\sqrt{3}$

Γ3. $\sigma\upsilon\nu(\vec{AB},\vec{AM}) = \frac{\vec{AB}\cdot\vec{AM}}{|\vec{AB}|\cdot|\vec{AM}|} = \frac{\vec{\beta}^2-2\vec{\alpha}^2+\vec{\alpha}\vec{\beta}}{\sqrt{3}\cdot 2\sqrt{3}} = \frac{4-2+1}{6} = \frac{1}{2}$

άρα $\varphi = 60^\circ$.