

5) α) Έίχαν  $\det(\vec{a}, \vec{b}) = 3$   $\det(\vec{b}, \vec{\gamma}) = -6$   $\det(\vec{a}, \vec{\gamma}) = 3 \neq 0$

β) Έίχαν  $\vec{\gamma} = k\vec{a} + \lambda\vec{b} \Rightarrow \vec{\gamma} = (k+2\lambda, k+5\lambda) \Rightarrow$

$$\left. \begin{array}{l} k+2\lambda = 4 \\ k+5\lambda = 7 \end{array} \right\} \Rightarrow \lambda = 1, k = 2 \text{ άρα } \vec{\gamma} = 2\vec{a} + \vec{b}$$

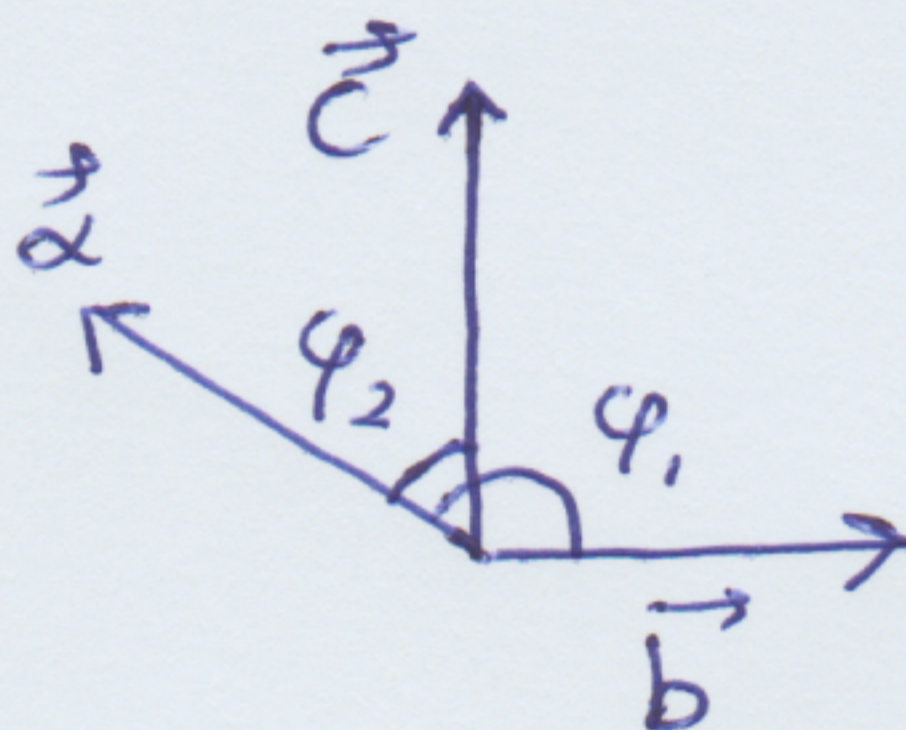
γ)  $\cos(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{7}{\sqrt{2} \cdot \sqrt{29}}$   $\cos(\vec{a}, \vec{\gamma}) = \frac{\vec{a} \cdot \vec{\gamma}}{|\vec{a}| \cdot |\vec{\gamma}|} = \frac{11}{\sqrt{2} \cdot \sqrt{65}}$

άρα  $\sqrt{29} \cdot \frac{7}{\sqrt{2} \cdot \sqrt{29}} + \frac{11}{\sqrt{2} \cdot \sqrt{65}} = 9\sqrt{2} \Rightarrow \frac{18}{\sqrt{2}} = 9\sqrt{2}$ , άρα άρα.

6) Έίχαν άρα  $|\vec{b}| = |\vec{c}| = \rho, \rho > 0, \vec{b} \cdot \vec{c} = 0$  (άρα  $\vec{b} \perp \vec{c}$ )

Έίχαν:  $(\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 \cdot \rho^2 \cdot \cos^2 \varphi_1$

$(\vec{a} \cdot \vec{c})^2 = |\vec{a}|^2 \cdot \rho^2 \cdot \cos^2 \varphi_2$



Έίχαν  $\varphi_1 - \varphi_2 = 90^\circ \Rightarrow$

$\varphi_1 = 90^\circ + \varphi_2 \Rightarrow \cos \varphi_1 = \cos(90^\circ + \varphi_2) \Rightarrow$

$\cos \varphi_1 = -\sin \varphi_2$  άρα,  $\cos^2 \varphi_1 = \sin^2 \varphi_2$ .

Συμμενίσ:  $(\vec{a} \cdot \vec{b})^2 + (\vec{a} \cdot \vec{c})^2 = |\vec{a}|^2 \cdot \rho^2 \cdot \cos^2 \varphi_1 + |\vec{a}|^2 \cdot \rho^2 \cdot \cos^2 \varphi_2$

$= |\vec{a}|^2 \cdot \rho^2 (\cos^2 \varphi_1 + \cos^2 \varphi_2) = |\vec{a}|^2 \cdot \rho^2 (\sin^2 \varphi_2 + \cos^2 \varphi_2) =$

$|\vec{a}|^2 \cdot \rho^2 = |\vec{a}|^2 \cdot |\vec{b}|^2$