

Επίσης,  $A_{f \circ f} = \{x \in [0, 4] \text{ ώστε } f(x) \in [0, 4]\} = \dots = [0, 4]$

$$y \in (f \circ f)(x) = f(f(x)) = f((\sqrt{x}-2)^2) = (\sqrt{(\sqrt{x}-2)^2} - 2)^2 \Rightarrow$$

$$f(f(x)) = (|\sqrt{x}-2| - 2)^2, \text{ όμως, αφού } x \in [0, 4], \sqrt{x}-2 \leq 0$$

επομένως  $f(f(x)) = (-\sqrt{x}+2-2)^2 = x.$

Επίσης, αν  $y = f(x) \Rightarrow y = (\sqrt{x}-2)^2 \Rightarrow |\sqrt{x}-2| = \sqrt{y} \Rightarrow$

$$-\sqrt{x}+2 = \sqrt{y} \Rightarrow \sqrt{x} = 2-\sqrt{y} \Rightarrow x = (2-\sqrt{y})^2 = (\sqrt{y}-2)^2$$

δηλαδή  $f^{-1}(x) = (\sqrt{x}-2)^2 = f(x).$

Γ3.  $(f \circ f)(x^2) = f^{-1}(x) \Rightarrow x^2 = (\sqrt{x}-2)^2 \Rightarrow (x-\sqrt{x}+2)(x+\sqrt{x}-2) = 0$

$$\Rightarrow \dots \Rightarrow x = 1$$

Γ4. i.  $\lim_{x \rightarrow 9} \frac{\sqrt[3]{x-1} + \sqrt{x} - 5}{(\sqrt{x}-2)^2 - 1} = \lim_{x \rightarrow 9} \frac{\sqrt[3]{x-1} - 2 + \sqrt{x} - 3}{(\sqrt{x}-3)(\sqrt{x}-1)} \stackrel{=: (x-9)}{=}$

$$\lim_{x \rightarrow 9} \frac{\frac{\sqrt[3]{x-1}-2}{x-9} + \frac{\sqrt{x}-3}{x-9}}{\frac{(\sqrt{x}-3)(\sqrt{x}-1)}{x-9}} = \lim_{x \rightarrow 9} \frac{\frac{1}{(\sqrt[3]{(x-1)^2+2\sqrt[3]{x-1}+4)} + \frac{1}{\sqrt{x}+3}}{\frac{\sqrt{x}-1}{\sqrt{x}+3}} =$$

$$= \frac{\frac{1}{12} + \frac{1}{6}}{\frac{2}{6}} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4},$$

ii)  $\lim_{x \rightarrow 4} \frac{n\mu(\sqrt{x}-2)^2}{x-4} = \lim_{x \rightarrow 4} \frac{n\mu(\sqrt{x}-2)^2}{(\sqrt{x}-2)(\sqrt{x}+2)} = (y = \sqrt{x}-2)$

$$\lim_{y \rightarrow 0} \frac{n\mu y^2}{y \cdot (y+4)} = \lim_{y \rightarrow 0} \frac{n\mu y^2}{y^2} \cdot \frac{y}{y+4} = 0.$$

Γ5. Αν  $y(t) = 1 \Rightarrow (\sqrt{x}-2)^2 = 1 \Rightarrow \sqrt{x}-2 = 1$  ή  $\sqrt{x}-2 = -1 \Rightarrow$   
 $x = 9$  ή  $x = 1$ , Δεχθεί  $x = 1$ . Συνεπώς  $x(t_0) = 1, y(t_0) = 1, x'(t_0) = 1$ .

$$y(t) = (\sqrt{x(t)} - 2)^2 \text{ άρα } y'(t) = 2(\sqrt{x(t)} - 2) \cdot \frac{1}{2\sqrt{x(t)}} \cdot x'(t) = \Rightarrow y'(t_0) = -1.$$