

## ΑΣΚΗΣΕΙΣ ΣΤΗΝ ΠΑΡΑΓΟΝΤΟΠΟΙΗΣΗ

### A. Απλώς βγάλτε κοινό παράγοντα:

1.  $x^2 - x =$

2.  $x^2 + 2x =$

3.  $2x - x^2 =$

4.  $4x^2 - 8x =$

5.  $2x - 2 =$

6.  $4x + 2 =$

7.  $-3x + 6 =$

8.  $-5x - 25 =$

9.  $-5x + 5 =$

10.  $4y + 4 =$

11.  $14x - 4 =$

12.  $24x + 16 =$

13.  $4x^3 + 2x^2 =$

14.  $8x^2 - 12x^3 =$

15.  $6x^2y - 3xy^2 =$

16.  $4a^3b^2 + 2ab^3 - ab^2 =$

17.  $3xy^3 - 6x^2y^2 + xy^2 =$

18.  $-3x^3y^2 + 9x^4y^4 =$

### B. Συνεχίζουμε με κοινό παράγοντα:

1.  $6x^3(x - a) - 3x^2(x - a) =$

5.  $4x(x - 2) - 2(2 - x) =$

2.  $2xy(x - 2)^2 - 4x^2y(x - 2)^3 =$

6.  $2x(x + 1) - 7x - 7 =$

3.  $4x(x + 3)^2 - 6x^2(x + 3) =$

7.  $3x(2x - 1) - 2x + 1 =$

4.  $2y(2y - 1) + 3(1 - 2y) =$

8.  $4x(2 - 3x) + 4 - 6x =$

### Γ. Ομαδοποιείστε τις παρακάτω ποσότητες (σε ζεύγη):

1.  $3ax - 6a + x - 2 =$

5.  $3a^2x - 6a^2 - 2yx + 4y =$

2.  $2xy - 4ay - x + 2a =$

6.  $2a^3x^2 + 4ay^2 + 12xy^2 + 6a^2x^3 =$

3.  $2a^2 - 6a - ax + 3x =$

7.  $3x + 2ax - 3a - 2x^2 =$

4.  $ax^2 + a^2x - 3x - 3a =$

8.  $2x(x - 3) - 3a(x - 3) - 4x + 6a =$

### Δ. Διαφορές τετραγώνων, από τις απλούστερες προς τις δυσκολότερες:

1.  $4x^2 - 9 =$

2.  $16 - 9a^2 =$

3.  $a^2x^4 - 25 =$

4.  $a^4 - 4x^6 =$

5.  $x^4 - a^4 =$

6.  $a^2 - 4x^4 =$

7.  $(3x - 2)^2 - 25 =$

8.  $(2a + 1)^2 - (1 - 3a)^2 =$

9.  $(x - 5)^2 - 4x^2 =$

10.  $(x + 5)^2 - (1 - 3x)^2 =$

11.  $16x^2 - (2x - 3)^2 =$

12.  $(7 - 2x)^2 - (3x - 2)^2 =$

13.  $4x^2 - (3x + 1)^2 =$

14.  $4(2x - 1)^2 - 9(x + 2)^2 =$

15.  $4ax^2 - 9a =$

16.  $2a^3x^4 - 8ax^8 =$

17.  $4a^2(x - 2) - 9(x - 2) =$

18.  $2x^4(x - 1) + 8x^2(1 - x) =$

Ε. Να γράψετε τις παρακάτω παραστάσεις σαν τετράγωνο διωνύμων:

1.  $4x^2 + 1 + 4x =$

2.  $25 + 10x^2 + x^4 =$

3.  $9a^2 + 12ax^2 + 4x^4 =$

4.  $a^2 + \frac{9}{a^2} + 6 =$

5.  $4x^2 + 9y^2 + 12xy =$

6.  $a^2x^4 + y^2 + 2ayx^2 =$

7.  $y^6 + 8y^3x^2 + 16x^4 =$

8.  $2x + 1 + x^2 =$

9.  $6a^3x^3 + 9a^4x^2 + a^2x^4 =$

10.  $4xy^2 + x^2 + 4y^4 =$

11.  $x^4 + 10x^2 + 25 =$

12.  $9a^2y^2 + 12ay^3 + 4y^4 =$

ΣΤ. Έχουν κοινό παράγοντα, αλλά υπάρχει και συνέχεια:

1.  $4x^2(x-3) - x + 3 =$

6.  $16(x-3)^2 - 4a^2(3-x)^2 =$

2.  $9(2x-1) - 2a^2x + a^2 =$

7.  $x^2(x-2) + 4x(2-x) + 4x - 8 =$

3.  $x^2(a-2) - 16(a-2) =$

8.  $x^3(x+2) - 4x(x+2) + ax^2 + 4ax + 4a =$

4.  $x^2(a+1) - 4x(a+1) + 4(a+1) =$

9.  $x^2(2x-1) - 3x(2x-1) + 4x - 2 =$

5.  $a^2(x-2) + 4ay(x-2) + 4y^2(x-2) =$

10.  $x^2(2a-3)^3 - 4x(2a-3)^2 + 8a - 12 =$

Ζ. Ομαδοποίηση, προτιμήστε το τρεις - ένας και εφαρμόστε ταυτότητες:

1.  $x^2 + 2x + 1 - a^2 =$

7.  $9x^2 - 9 - y^2 - 6y =$

2.  $x^2 - 2ax + a^2 - 16 =$

8.  $2x + 4a^2 - 1 - x^2 =$

3.  $1 - x^2 + 2ax - a^2 =$

9.  $4x^2 + 12x + 9 - y^2 + 2ay - 1 =$

4.  $9a^2 - 4x^2 - 4x - 1 =$

10.  $y^2 + \frac{4}{y^2} - 5 =$

5.  $25x^4 + 10x^2 + 1 - 9x^6 =$

11.  $27a^3 - 12ax^2 - 12ax - 3a =$

6.  $x^2 + \frac{1}{x^2} + 1 =$

12.  $x + \frac{1}{x} - 3 =$

Η. Διασπάστε κατάλληλα έναν από τους τρεις όρους και παραγοντοποιήστε:

1.  $x^2 - 5x + 6 =$

2.  $x^2 - 5xy + 4y^2 =$

3.  $x^2 + 3x + 2 =$

4.  $4x^2 - 7xy + 3y^2 =$

5.  $x^2 - 4x - 5 =$

6.  $3a^2 - ay - 2y^2 =$

7.  $x^2 + 3ax - 4a^2 =$

8.  $x^2 + ax - 12a^2 =$

9.  $4x^2 + 8xy + 3y^2 =$

10.  $4a^2 - 4ax - 3x^2 =$

Θ. Δοκιμάστε να τις παραγοντοποιήσετε, αφήνοντας και τη φαντασία σας ελεύθερη:

1.  $(x - 2a)^2(x - 2) - 4x^3 + 8x^2 =$
2.  $3a(2x + 1) - 2y(2x + 1) - 2yx^2 + 3ax^2 =$
3.  $x^2 - y^2 + 4x - 6y - 5 =$
4.  $4x^2 - 9y^2 - 8x - 6y + 3 =$
5.  $3a^2x - (3x + 2)(4x^2 - 12x + 9) + 2a^2 =$
6.  $(x^2 - 9)^2 - 4(x + 3)^2 =$
7.  $3(x - 2)(x^2 - 9) + 7(2 - x)(3 - x)^2 =$
8.  $25x^2 - 9y^2 - 10x - 12y + 3 =$
9.  $x^2y^2(z^4 + 1) - (x^4 + y^4)z^2 =$
10.  $x^2(y + z) + y^2(x + z) + z^2(x + y) + 2xyz =$
11.  $x^2(y - z) + y^2(z - x) + z^2(x - y) =$
12.  $xy(x - y) + yz(y - z) + xz(z - x) =$

Ι. Μερικές ασκήσεις με τις «μυστικές» ταυτότητες (Άθροισμα και διαφορά κύβων):

1.  $8x^3 - 27 =$
2.  $a^3 + 8 =$
3.  $27 + x^6 =$
4.  $(2x - 1)^3 + a^3x^3 =$
5.  $(1 + 2x)^3 - 8y^3 =$
6.  $(x + 2)^3 + (1 - x)^3 =$
7.  $(2x + 3)^3 - (x - 1)^3 =$
8.  $x^2(x^2 - x + 1) + x^4 + x =$
9.  $x^3 - yx^2 + xy - y + 1 =$
10.  $x^3(y - z) + y^3(z - x) + z^3(x - y) =$

ΑΠΑΝΤΗΣΕΙΣ:

A. 1.  $x(x-1)$       2.  $x(x+2)$       3.  $x(2-x)$       4.  $4x(x-2)$       5.  $2(x-1)$   
 6.  $2(2x+1)$       7.  $-3(x-2)$       8.  $-5(x+5)$       9.  $-5(x-1)$       10.  $4(y+1)$   
 11.  $2(7x-2)$       12.  $8(3x+4)$       13.  $2x^2(2x+1)$       14.  $4x^2(2-3x)$       15.  $3xy(2x-y)$   
 16.  $ab^2(4a^2+2b-1)$       17.  $xy^2(3y-2xy+1)$       18.  $-3x^3y^2(1-3xy^2)$

B. 1.  $3x^2(x-a)(2x-1)$       2.  $2xy(x-2)^2[1-2x(x-2)] = 2xy(x-2)^2(1-2x^2+4x)$   
 3.  $2x(x+3)[2(x+3)-3x] = 2x(x+3)(6-x)$       4.  $(2y-1)(2y-3)$   
 5.  $2(x-2)(2x+1)$       6.  $2x(x+1)-7(x+1) = (x+1)(2x-7)$   
 7.  $(2x-1)(3x-1)$       8.  $4x(2-3x)+2(2-3x) = 2(2-3x)(2x+1)$

Γ. 1.  $3a(x-2)+(x-2) = (x-2)(3a+1)$       2.  $2y(x-2a)-(x-2a) = (x-2a)(2y-1)$   
 3.  $2a(a-3)-x(a-3) = (a-3)(2a-x)$       4.  $ax(x+a)-3(x+a) = (x+a)(ax-3)$   
 5.  $3a^2(x-2)-2y(x-2) = (x-2)(3a^2-2y)$   
 6.  $2a^2x^2(a+3x)+4y^2(a+3x) = 2(a+3x)(a^2x^2+2y^2)$   
 7.  $x(3-2x)-a(3-2x) = (3-2x)(x-a)$   
 8.  $(x-3)(2x-3a)-2(2x-3a) = (2x-3a)(x-5)$

Δ. 1.  $4x^2-9 = (2x-3)(2x+3)$       2.  $16-9a^2 = (4-3a)(4+3a)$   
 3.  $a^2x^4-25 = (ax^2+5)(ax^2-5)$       4.  $a^4-4x^6 = (a^2-2x^3)(a^2+2x^3)$   
 5.  $x^4-a^4 = (x^2+a^2)(x-a)(x+a)$       6.  $a^2-4x^4 =$   
 7.  $(3x-2)^2-25 = 3(3x-7)(x+1)$       8.  $(2a+1)^2-(1-3a)^2 = 5a(2-a)$   
 9.  $(x-5)^2-4x^2 = -(x+5)(3x-5)$       10.  $(x+5)^2-(1-3x)^2 = 8(x+1)(3-x)$   
 11.  $16x^2-(2x-3)^2 = 3(2x-1)(2x+3)$       12.  $(7-2x)^2-(3x-2)^2 = (9-5x)(x+5)$   
 13.  $4x^2-(3x+1)^2 = -(5x+1)(x+1)$       14.  $4(2x-1)^2-9(x+2)^2 = (x-8)(7x+4)$   
 15.  $4ax^2-9a = a(2x-3)(2x+3)$       16.  $2a^3x^4-8ax^8 = 2ax^4(a-2x^2)(a+2x^2)$   
 17.  $4a^2(x-2)-9(x-2) = (x-2)(2a-3)(2a+3)$   
 18.  $2x^4(x-1)+8x^2(1-x) = 2x^2(x-1)(x-2)(x+2)$

$$1. 4x^2 + 1 + 4x = (2x + 1)^2$$

$$2. 25 + 10x^2 + x^4 = (5 + x^2)^2$$

$$3. 9a^2 + 12ax^2 + 4x^4 = (3a + 2x^2)^2$$

$$4. a^2 + \frac{9}{a^2} + 6 = \left(a + \frac{3}{a}\right)^2$$

$$E. 5. 4x^2 + 9y^2 + 12xy = (2x + 3y)^2$$

$$6. a^2x^4 + y^2 + 2axy^2 = (ax^2 + y)^2$$

$$7. y^6 + 8y^3x^2 + 16x^4 = (y^3 + 4x^2)^2$$

$$8. 2x + 1 + x^2 = (x + 1)^2$$

$$9. 6a^3x^3 + 9a^4x^2 + a^2x^4 = (3a^2x + ax^2)^2$$

$$10. 4xy^2 + x^2 + 4y^4 = (x + 2y^2)^2$$

$$11. x^4 + 10x^2 + 25 = (x^2 + 5)^2$$

$$12. 9a^2y^2 + 12ay^3 + 4y^4 = (3ay + 2y^2)^2$$

ΣΤ.

$$1. 4x^2(x - 3) - x + 3 = (x - 3)(2x - 1)(2x + 1)$$

$$2. 9(2x - 1) - 2a^2x + a^2 = 9(2x - 1) - a^2(2x - 1) = (2x - 1)(3 - a)(3 + a)$$

$$3. x^2(a - 2) - 16(a - 2) = (a - 2)(x - 4)(x + 4)$$

$$4. x^2(a + 1) - 4x(a + 1) + 4(a + 1) = (a + 1)(x - 2)^2$$

$$5. a^2(x - 2) + 4ay(x - 2) + 4y^2(x - 2) = (x - 2)(a + 2y)^2$$

$$6. 16(x - 3)^2 - 4a^2(3 - x)^2 = 4(x - 3)^2(2 - a)(2 + a)$$

$$7. x^2(x - 2) + 4x(2 - x) + 4x - 8 = x^2(x - 2) - 4x(x - 2) + 4(x - 2) = (x - 2)^3$$

$$8. x^3(x + 2) - 4x(x + 2) + ax^2 + 4ax + 4a = x^3(x + 2) - 4x(x + 2) + a(x + 2)^2 =$$

$$x(x + 2)(x + 2)(x - 2) + a(x + 2)^2 = (x + 2)^2(x^2 - 2x + a)$$

$$9. x^2(2x - 1) - 3x(2x - 1) + 4x - 2 = (2x - 1)(x - 1)(x - 2)$$

$$10. x^2(2a - 3)^3 - 4x(2a - 3)^2 + 8a - 12 = (2a - 3)(2ax - 3x + 2)^2$$

Z.

$$1. x^2 + 2x + 1 - a^2 = (x+1)^2 - a^2 = (x+1+a)(x+1-a)$$

$$2. x^2 - 2ax + a^2 - 16 = (x-a)^2 - 16 = (x-a+4)(x-a-4)$$

$$3. 1 - x^2 + 2ax - a^2 = 1 - (x-a)^2 = (1-x+a)(1+x-a)$$

$$4. 9a^2 - 4x^2 - 4x - 1 = 9a^2 - (2x+1)^2 = (3a+2x+1)(3a-2x-1)$$

$$5. 25x^4 + 10x^2 + 1 - 9x^6 = (5x^2+1)^2 - 9x^6 = (5x^2+1-3x^3)(5x^2+1+3x^3)$$

$$6. x^2 + \frac{1}{x^2} + 1 = x^2 + \frac{1}{x^2} + 2 - 1 = \left(x + \frac{1}{x}\right)^2 - 1 = \left(x + \frac{1}{x} + 1\right)\left(x + \frac{1}{x} - 1\right)$$

$$7. 9x^2 - 9 - y^2 - 6y = 9x^2 - (3+y)^2 = (3x+3+y)(3x-3-y)$$

$$8. 2x + 4a^2 - 1 - x^2 = 4a^2 - (x-1)^2 = (2a+x-1)(2a-x+1)$$

$$9. 4x^2 + 12x + 9 - y^2 + 2y - 1 = (2x+3)^2 - (y-1)^2 = (2x+y+2)(2x-y+4)$$

$$10. y^2 + \frac{4}{y^2} - 5 = \left(y - \frac{2}{y}\right)^2 - 1 = \left(y - \frac{2}{y} - 1\right)\left(y - \frac{2}{y} + 1\right)$$

$$11. 27a^3 - 12ax^2 - 12ax - 3a = 3a(9a^2 - 4x^2 - 4x - 1) = 3a[9a^2 - (2x+1)^2] = 3a(3a-2x-1)(3a+2x+1)$$

$$12. x + \frac{1}{x} - 3 = x + \frac{1}{x} - 2 - 1 = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 - 1 = \left(\sqrt{x} - \frac{1}{\sqrt{x}} + 1\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}} - 1\right)$$

$$1. x^2 - 5x + 6 = (x-2)(x-3) \quad 2. x^2 - 5xy + 4y^2 = (x-y)(x-4y)$$

$$3. x^2 + 3x + 2 = (x+1)(x+2) \quad 4. 4x^2 - 7xy + 3y^2 = ($$

$$H. 5. x^2 - 4x - 5 = (x+1)(x-5) \quad 6. 3a^2 - ay - 2y^2 =$$

$$7. x^2 + 3ax - 4a^2 = (x-a)(x+4a) \quad 8. x^2 + ax - 12a^2 =$$

$$9. 4x^2 + 8xy + 3y^2 = \quad 10. 4a^2 - 4ax - 3x^2 =$$