

2.1 1. $15a^3x - 15a^3y + 5a^2\beta x - 5a^2\beta y = 15a^3(x - y) + 5a^2\beta(x - y) = 5a^2(x - y)(3a + \beta)$

2. $a^4x^2 - a^3x^3 - a^3x + a^2x^2 = a^2x(a^2x - ax^2 - a + x) = a^2x(ax - 1)(a - x)$

3. $x^2 - 4xy^2 - 16 + 4y^4 = (x - 2y^2)^2 - 16 = (x - 2y^2 - 4)(x - 2y^2 + 4)$

4. $4x^2 - 9y^2 - 6yz^2 - z^4 = 4x^2 - (3y + z^2)^2 = (2x + 3y + z^2)(2x - 3y - z^2)$

5. $4a^2x^2 - 4b^2x^2 - 9a^2y^2 + 9b^2y^2 = 4x^2(a^2 - b^2) - 9y^2(a^2 - b^2) = (2x - 3y)(2x + 3y)(a - b)(a + b)$

6. $3a^2x^2 - 3b^2x^2 + 6bx^3 - 3x^4 = 3\left[a^2x^2 - (bx - x^2)^2\right] = 3x^2(a - b + x)(a + b - x)$

7. $4xy(x - y) - 6x(x - y)^2 + 2x(x^2 - y^2) = 2x(x - y)(2y - 3x + 3y + x + y) = 4x(x - y)(3y - x)$

8. $x^2 - y^2 + 4x - 2y + 3 = (x + 2)^2 - (y + 1)^2 = (x + y + 3)(x - y + 1)$

9. $x^2(y - w) + y^2(w - x) + w^2(x - y) = x^2y - x^2w + y^2w - y^2x + w^2(x - y) =$
 $= xy(x - y) - w(x - y)(x + y) + w^2(x - y) = (x - y)(xy - wx - wy + w^2) = (x - y)[y(x - w) - w(x - w)] =$
 $(x - y)((x - w)(y - w))$

10. $(\alpha\chi + \mu\beta\psi)^2 - \mu(\alpha\psi + \beta\chi)^2 = \alpha^2\chi^2 + 2\alpha\chi\mu\beta\psi + \mu^2\beta^2\psi^2 - \mu\alpha^2\psi^2 - 2\alpha\beta\chi\mu\psi - \mu\beta^2\chi^2 =$
 $= \alpha^2(\chi^2 - \mu\psi^2) - \mu\beta^2(\chi^2 - \mu\psi^2) = (\chi^2 - \mu\psi^2)(\alpha^2 - \mu\beta^2)$

2.2 Να απλοποιήσετε - αφού πρώτα κάνετε ομώνυμα - τα παρακάτω κλάσματα:

1. $\frac{2(\alpha - \beta)}{\alpha^3 + \alpha^2\beta} + \frac{\alpha + \beta}{\alpha^3 + \alpha\beta^2 + 2\alpha^2\beta} - \frac{5}{\alpha^2 + \alpha\beta} = \frac{2(\alpha - \beta)}{\alpha^2(\alpha + \beta)} + \frac{\alpha + \beta}{\alpha(\alpha + \beta)^2} - \frac{5}{\alpha(\alpha + \beta)} =$
 $= \frac{2\alpha - 2\beta + \alpha - 5\alpha}{\alpha^2(\alpha + \beta)} = \frac{-2(\alpha + \beta)}{\alpha^2(\alpha + \beta)} = -\frac{2}{\alpha^2}$

2. $\frac{3\alpha + \beta}{2\alpha^2 - 3\alpha\beta} - \frac{2\alpha + \beta}{3\alpha\beta - 2\alpha^2} + \frac{12\alpha + 10\beta}{9\beta^2 - 4\alpha^2} = \frac{3\alpha + \beta}{\alpha(2\alpha - 3\beta)} + \frac{2\alpha + \beta}{\alpha(2\alpha - 3\beta)} - \frac{12\alpha + 10\beta}{(2\alpha - 3\beta)(2\alpha + 3\beta)} =$
 $\frac{(5\alpha + 2\beta)(2\alpha + 3\beta) - 12\alpha^2 - 10\alpha\beta}{\alpha(2\alpha - 3\beta)(2\alpha + 3\beta)} = \frac{-2\alpha^2 + 9\alpha\beta + 6\beta^2}{\alpha(2\alpha - 3\beta)(2\alpha + 3\beta)}$

3. $\frac{1}{x - y} + \frac{2x + y}{x^2 + xy} - \frac{2x - y}{xy - y^2} = \frac{1}{x - y} + \frac{2x + y}{x(x + y)} - \frac{2x - y}{y(x - y)} = \frac{xy(x + y) + y(2x + y)(x - y) - x(x + y)(2x - y)}{xy(x - y)(x + y)} =$
 $\frac{x^2y + xy^2 + (2xy + y^2)(x - y) - (x^2 + xy)(2x - y)}{xy(x - y)(x + y)} = \frac{x^2y + xy^2 + 2x^3 - xy^2 - y^3 - 2x^3 + xy^2 - x^2y}{xy(x - y)(x + y)} =$
 $\frac{y^2(x - y)}{xy(x - y)(x + y)} = \frac{y}{x(x + y)}$

4. $\frac{x - 3}{3x^2 + x} - \frac{x + 3}{x - 3x^2} - \frac{x}{9x^2 - 1} + \frac{4x^2 - 7}{9x^3 - x} = \frac{x - 3}{x(3x + 1)} + \frac{x + 3}{x(3x - 1)} - \frac{x}{(3x - 1)(3x + 1)} + \frac{4x^2 - 7}{x(3x - 1)(3x + 1)} =$
 $\frac{(x - 3)(3x - 1) + (x + 3)(3x + 1) - x^2 + 4x^2 - 7}{x(3x - 1)(3x + 1)} = \frac{9x^2 - 1}{x(3x - 1)(3x + 1)} = \frac{1}{x}$

2.3 Εκτελούμε κάθε φορά τις πράξεις στο 1^ο και το 2^ο μέλος και προκύπτουν τα ίδια αποτελέσματα.

2.4 Αν ισχύουν οι σχέσεις: $a+b=3$, $ab=-2$, να βρείτε τις τιμές των παραστάσεων:

$$a^2 + b^2 = (a+b)^2 - 2ab = 9 + 4 = 13, \quad a^3 + b^3 = (a+b)(a^2 - ab + b^2) = 3(13 + 2) = 45$$

$$\frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = -\frac{13}{2}, \quad \frac{b}{a^2} + \frac{a}{b^2} = \frac{b^3 + a^3}{a^2 b^2} = \frac{45}{4}, \quad a^2 b + ab^2 = ab(a+b) = -6$$

$$x^2 + y^2 + z^2 = xy + yz + zx \Leftrightarrow 2x^2 + 2y^2 + 2z^2 = 2xy + 2yz + 2zx \Leftrightarrow$$

$$2.5 \quad x^2 + x^2 + y^2 + y^2 + z^2 + z^2 - 2xy - 2yz - 2zx = 0 \Leftrightarrow (x-y)^2 + (y-z)^2 + (z-x)^2 = 0$$

$$\Leftrightarrow x-y=0 \text{ και } y-z=0 \text{ και } z-x=0 \Leftrightarrow x=y=z$$

2.6

$$\text{Αν } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0 \Leftrightarrow yz + xz + xy = 0, \text{ οπότε: } \frac{yz}{x^2} + \frac{xz}{y^2} + \frac{xy}{z^2} = -\frac{z+y}{x} - \frac{z+x}{y} - \frac{x+y}{z} =$$

$$-\frac{z^2 y + zy^2 + z^2 x + zx^2 + x^2 y + xy^2}{xyz} = -\frac{z^2(x+y) + xy(x+y) + z(x^2 + y^2)}{xzy} =$$

$$-\frac{(x+y)(z^2 + xy) + z[(x+y)^2 - 2xy]}{xyz} = -\frac{(x+y)(z^2 - yz - zx) + z(x+y)^2 - 2xyz}{xyz} =$$

$$-\frac{z(x+y)(z-y-x+x+y) - 2xyz}{xyz} = -\frac{-xyz - 2xyz}{xyz} = 3$$

2.7

$$1. \frac{x^{-2} \cdot (y^{-1} \cdot \frac{1}{x^{-3}})^{-2}}{\left(x^{-3} \cdot \frac{1}{y^{-2}}\right)^{-1} \cdot y^{-7}} = \frac{x^{-2} \cdot y^2 \cdot x^{-6}}{x^3 \cdot y^{-2} \cdot y^{-7}} = x^{-11} \cdot y^{11} = (-2018)^{-11} \cdot 2018^{11} = -1$$

$$2. \frac{(x^3 : y^{-2})^{-1} \cdot \left(\frac{x^{-2}}{y^3} \cdot \frac{1}{x}\right)^{-2}}{\left(\frac{x^{-3}}{y^{-1}} : \frac{x^2}{y^3}\right)^{-1} \cdot x^{-10}} = \frac{x^{-3} \cdot y^{-2} \cdot x^6 \cdot y^6}{x^5 \cdot y^{-4} \cdot x^{-10}} = (xy)^8 = \left(\frac{9}{2} \cdot \frac{2}{9}\right)^8 = 1 \quad (\alpha\text{φο}\acute{\upsilon} \ x=4,5 \ y=0,\bar{2})$$

$$2.8 \quad \alpha = 2 \cdot 3^{v+1} + 3^{v+2} = 2 \cdot 3 \cdot 3^v + 3^2 \cdot 3^v = 15 \cdot 3^v = 5 \cdot 3^{v+1} = \text{πολ / σιο του } 5$$

$$\beta = 10 \cdot 5^{v-1} + 3 \cdot 5^{v+1} + 5^{v+2} = 10 \cdot \frac{5^v}{5} + 3 \cdot 5^v \cdot 5 + 5^v \cdot 5^2 = 42 \cdot 5^v = \text{πολ / σιο του } 21$$

$$2.9 \quad \alpha. (2019 - 2018)^2 = 1 \quad \beta. 2 + 2 = 4$$

$$2.10 \quad \alpha. (-3, 3) \quad \beta. (-1, 1] \quad \gamma. \left(-8, -\frac{1}{2}\right] \quad \delta. (-8, -3) \quad \epsilon. \left(-5, -\frac{13}{2}\right) \quad \phi. [4, 64]$$

2.11

a. $x^2 + xy + y^2 \geq 0 \Leftrightarrow 2x^2 + 2xy + 2y^2 \geq 0 \Leftrightarrow (x+y)^2 + x^2 + y^2 \geq 0$

b. $x^2 - xy + y^2 \geq 0 \Leftrightarrow \dots \Leftrightarrow (x-y)^2 + x^2 + y^2 \geq 0$

c. $x + \frac{1}{x} \geq 2 \Leftrightarrow x^2 + 1 - 2x \geq 0 \Leftrightarrow (x-1)^2 \geq 0$

d. $\frac{2x}{x^2+1} \geq -1 \Leftrightarrow 0 \geq -x^2 - 1 - 2x \Leftrightarrow (x+1)^2 \geq 0$

2.12

a. $x^2 + 2x + y^2 + 1 \leq 0 \Leftrightarrow (x+1)^2 + y^2 \leq 0 \Leftrightarrow x = -1 \text{ και } y = 0$

b. $x^2 - x + \frac{1}{4} + y^2 \leq 0 \Leftrightarrow \left(x - \frac{1}{2}\right)^2 + y^2 \leq 0 \Leftrightarrow x = \frac{1}{2} \text{ και } y = 0$

c. $x^2 + y^2 - 6x + 4y + 13 = 0 \Leftrightarrow (x-3)^2 + (y+2)^2 = 0 \Leftrightarrow x = 3 \text{ και } y = -2$

2.13

$x + y > 2 + \frac{xy}{2} \Leftrightarrow 2x + 2y - 4 - xy > 0 \Leftrightarrow 2(x-2) - y(x-2) > 0 \Leftrightarrow$
 $(x-2)(2-y) > 0$, που ισχύει γιατί $x < 2 < y$.

2.14 $3^{57} = (3^3)^{19} = 27^{19}$, ενώ $5^{38} = (5^2)^{19} = 25^{19}$, άρα $3^{57} > 5^{38}$

2.15 $a = 617^4 - 578^4 = (617 - 578)(617 + 578)(617^2 + 578^2) = 39 \cdot (617 + 578)(617^2 + 578^2)$